1. What distances can you be certain of on the ruler in Model 1? 

Only 0 and 10 are known for sure (rest are estimates) 

2. Six students used the ruler in Model 1 to measure the length of a metal strip. Their measurements are shown at the right. Were all of the students able to agree on a single value (1, 2, 3…) for any digit (ones place, tenths place, etc.) in the measurement? If yes, which value and digit did they agree on? 

They did not agree on any one digit. 

3. The ruler in Model 1 is not very useful, but a measurement can be estimated. Discuss in your group how each student might have divided up the ruler “by eye” in order to get the measurement that he or she recorded. 

Susan and Maya likely rounded to the nearest 1/10\textsuperscript{th} of the unit (1/10\textsuperscript{th} of 10 is 1), just viewed it differently. Jonah likely mentally divided it into quarters (2.5 is ¼ of 10). Tony, Emily, and Dionne likely put their own tick marks by eye at 1 each cm, and then estimated how much further it was 

4. The students obtained a better ruler, shown in Model 2. What distances can you be certain of on this ruler? 

All whole numbers from 0 to 10 are known for sure (rest are estimates) 

5. Were the students able to agree on a single value (1, 2, 3…) for any digit (ones place, tenths place, etc.) in their measurements using the ruler in Model 2? If yes, what value in what digit did they agree on? 

They all agreed on the first digit, the 3 (which is in the ones place). 

6. What feature of the ruler in Model 2 made it possible for the students to agree on a value in that digit? 

There were more marks shown so everyone could see the value was between 3 and 4 cm, and just had to estimate where in between.
7. There will always be uncertainty in any measurement. This causes variation in measurements even if people are using the same instrument. Compare the variation in the measurements made by the six students using the rulers in Models 1 and 2. Which ruler resulted in greater variation? Explain why that ruler caused more variation.

In model 1, the lowest estimate was 2 and the highest was 3.33, for a variance of 1.33cm, while for model 2, the lowest estimate was 3 and the highest was 3.3, for a variance of 0.3cm. The model 1 ruler caused the most variation because there were fewer divisions, and thus had the most estimation required.

8. The students obtained an even better ruler, shown above in Model 3.
   a. Were the students able to agree on a single value for any of the digits in their measurements using the new ruler? If yes, what value(s) did they agree on in which digits?

They all agreed on the first two digits, the 3.2 (which are the ones and tenths places).

   b. What feature of the ruler in Model 3 made it possible for the students to agree on the values in those digits?

Having the smaller divisions let them see the length was between the 3.2 and 3.3 marks, so all agreed on 3.2
Model 4 – Valid Measurements

9. The measurements taken in Models 1–3 have been combined in Model 4. The measurements that follow the rules of measurement agreed upon by scientists are in the “Valid Measurements” column. Those that do not follow the rules are in the “Invalid Measurements” column. For each valid measurement shown in Model 4, draw a square around the certain digits (if any) and circle the digits that were estimated (if any).

<table>
<thead>
<tr>
<th>Valid Measurements</th>
<th>Invalid Measurements</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 cm</td>
<td>2.5 cm</td>
</tr>
<tr>
<td>2 cm</td>
<td>3.00 cm</td>
</tr>
<tr>
<td>3/4 cm</td>
<td>3.33 cm</td>
</tr>
<tr>
<td>3.2 cm</td>
<td>3 cm</td>
</tr>
<tr>
<td>3.1 cm</td>
<td>3.25 cm</td>
</tr>
<tr>
<td>3.3 cm</td>
<td>3.20 cm</td>
</tr>
<tr>
<td>3.2 cm</td>
<td>3.2 cm</td>
</tr>
<tr>
<td>3.20 cm</td>
<td>3.215 cm</td>
</tr>
<tr>
<td>3.21 cm</td>
<td>3.205 cm</td>
</tr>
</tbody>
</table>

10. Based on the examples in Model 4, circle the best phrase to complete each sentence below.
   a. In a valid measurement, you record \( \) estimated digit(s).
   b. In a valid measurement, the estimated digit is the \( \) digit in the measurement.
   c. In a valid measurement, the estimated digit corresponds to \( \) on the instrument.

11. Using Ruler B from Model 4, Tony recorded a measurement of 3 cm. Explain why this was an invalid measurement.

   In ruler B, they are certain of the 3cm but need to estimate one more digit beyond the last certain digit

12. Using Ruler B from Model 4, Dionne recorded a measurement of 3.20 cm, which was invalid. But when Maya made the same measurement using Ruler C, it was considered valid. Explain why the zero was acceptable when using Ruler C, but not when using Ruler B.

   We can only estimate to the 1/10\( ^{th} \) of the smallest division shown, and that smallest unit is 1cm in B and 0.1cm in C. The 0 in 3.20 is in the 1/100\( ^{th} \) spot which is beyond the precision for B but within the precision possible for C.
13. A student recorded the length of a test tube as 5.0 cm. Which ruler in Model 4 was the student using? Explain.

We can only estimate to the 1/10\textsuperscript{th} of the smallest division shown, the 0 in the 5.0 must be the estimated digit, meaning the 1 cm marks were the only certain numbers, so the ruler must have been B.

14. In Model 4, Ricky recorded his measurement 3.19 cm using Ruler C. His classmates thought he was wrong because his second digit was not “2.” However, Ricky’s recorded measurement is perfectly valid. Explain.

Ricky’s 2\textsuperscript{nd} digit would still round to be 2, so rounded off all are still in agreement that it rounds to 3.2

15. Record the length of the wooden splint to the proper number of significant digits.

7.00 cm, the end is right at the 7 cm mark, and since the smallest divisions are 0.1 cm we need to add the estimated digit in the hundredths place.

16. Record the length of the wooden splint to the proper number of significant digits.

7.00 cm, the end is right at the 7 cm mark, and since the smallest divisions are 0.1 cm we need to add the estimated digit in the hundredths place.
**Extension Questions**

17. When using an electronic device, such as an electronic balance, the measurement displayed on the screen is assumed to have one estimated digit included. In fact, you’ll often see the estimated digit changing rapidly, because there is fluctuation in the estimate. Explain why it is important to record the zero in the measurement shown to the right.

Even though the 0 is estimated we need it to tell anyone seeing the data that we measured to the 0.01 place. If we only recorded the 1.2, we’d be telling anyone looking at our data that data was only recorded to the 0.1 place.

18. Consider a 1000-mL graduated cylinder with marks every 100 mL.
   
   a. A student records the volume of liquid in the cylinder as 750 mL. Is this a correct measurement? Explain.

   Yes, this is a correct, with 100mL smallest units, the estimated digit would be $1/10^{th}$ of 100mL, or to 10mL, and 750mL is expressed to the nearest 10mL.

   b. Are all of the digits in the described measurement of 750 mL significant? Explain.

   Only the 7 and the 5 are significant, the 7 is certain and the 5 was the estimated digit. There is no way, without finer divisions, to know how many ones there might be.

19. A student properly records the length of a block as 120 cm. Draw the markings on the ruler that was used to measure the block.

   ![Markings on the ruler](https://via.placeholder.com/150)

   1/10$^{th}$ of 100cm is 10cm and 120cm is measured to the nearest 10cm

   Side note, if they had included a decimal point, like 120. Or 1.20x$10^2$, then the 0 would have mattered (the decimal tells the reader that any 0s after the last non-zero number were measured or estimated)
Most increments are done by powers of 10 (we’re using metric after all), however, sometimes the increment is not a direct power of 10, such as 50, 40, 20, etc. With non-standard divisions the data must be recorded differently to show that. Often times the data is shown with a ± of 1/10\textsuperscript{th} the smallest unit such as 120cm ± 2cm would show the estimation was to 2cm, so the smallest increment would have been 20cm.