Unit Dimensional Analysis Activity

**Why?** In this activity we will see that it is possible to look at a situation from several points of view, or to take measurements of that same situation using different units of measure. Every measurement has 2 components: **magnitude** and **dimension**. **Magnitude** is the value of the number in the measurement and **dimension** is the unit of measure (e.g., grams, centimeters, inches or liters.)

- *If a measurement is given, can we convert that measurement to different units to meet our needs?*

**Model: Car Trip**
All of these are values associated with 1 car trip:
- 120 miles
- 100 minutes
- 5 gallons of gasoline
- $17.25 (price of gas)
- 1 bathroom break (assume breaks are all same length)
- 34 songs on your iPod® (assume all are same length)

**Group Instructions:** When addressing each question, one group member should be assigned the task of reading the question aloud for the rest of the group. The manager should rotate that role among group members throughout the assignment.

**Critical Questions:** (All values need either work or a short written explanation how you know)

1. How long does it take to drive 120 miles?

2. How long does it take to drive 240 miles?

3. How many miles can you drive on 5 gallons of gas?

4. How many miles can you drive on 1 gallon of gas?

5. Show how you solved question # 4. Be sure to include the units in your calculations.
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6. Show the miles per gallon as a fraction (ratio) with numerator and denominator. Which is the numerator? Which is the denominator?

7. Using a grammatically correct sentence describe how you made the choice for # 6.

8. Is there another way to write the fractional relationship of gallons and miles (one that doesn’t show miles per gallon)? Show this way.

9. Why might you want to write the ratio this 2nd way?

10. Here are 3 other ratio relationships that we can obtain from the model:

   \[
   \frac{1 \text{ bathroom break}}{120 \text{ miles}} \quad \frac{5 \text{ gallons}}{100 \text{ minutes}} \quad \frac{34 \text{ songs}}{17.25 \text{ of gas}}
   \]

   Write 4 other such relationships that you can obtain from the model, try to write ones that are not just inverses of ones already listed:

These relationships are called Conversion Factors.
List the components that are necessary in a conversion factor:

Using complete sentences consult with your group and come up with a description of a conversion factor. What are its essential components and what is its purpose?

Box is to show Key Ideas

11. Which one of the conversion factors from #10 would you use to determine how long it would take to burn 12 gallons of gas?
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12. Construct the conversion factor needed to determine how many songs you would hear in 500 miles. Do not solve yet (you’ll do that in 13).

13. Solve # 12 mathematically. Show your work below and be sure to include units.

Reflections:

14. As a group, write grammatically correct English sentences to describe the objective of the activity at this point. Be prepared to share your answer with the class.

15. After having shared with the class, does your group still agree with your initial assessment of what the objective is?

16. As a group, can you think of a situation when a scientist or chemist might need to use conversion factors to solve a problem? Give an example.

Exercises:

Using conversion factors to solve a problem is called Dimensional Analysis.

You should now be able to solve the following problems.

17. Solve this problem without using a calculator: \( \frac{6 \cdot 17 \cdot 3 \cdot 13}{13 \cdot 9 \cdot 17} = \)

18. Write a mathematical rule that makes this problem easier to solve (you likely used this rule).

19. Solve this problem:

\( \frac{\text{miles} \cdot \text{songs} \cdot \text{gallons}}{\text{miles} \cdot \text{gallons}} = \)
It is often convenient to represent calculations of this type as a “cancellation line.” The cancellation line for Problem 17 would look like this: \[
\frac{6}{1} \cancel{17} \frac{3}{1} \frac{12}{9} \cancel{47} = 2
\]
Cancellation lines can also be used with units.

Ex. Calculate the number of minutes in 5 days:
\[
\frac{5 \text{ days}}{1} \frac{24 \text{ hours}}{1 \text{ day}} \frac{60 \text{ minutes}}{1 \text{ day}} = 7,200 \text{ min}
\]

**Use a cancellation line** to solve the remaining problems.

20. How many miles would you have to drive to hear 43 songs? Show how you solve the problem using units and conversion factors.

21. Using your answer from # 20, how many minutes would this take? Again show how you solve the problem using units and conversion factors.

22. Show how you can combine problems # 20 and # 21 into one. Draw a line through any units that cancel. Put your answer on the board.

23. Write a grammatically correct English sentence to describe which unit you will be left with in the answer.

### On your own

24. The average human heart beats 72 beats/minute. If you live to be 80 years old, how many times does your heart beat. What conversion factors do you need to know to solve this problem. List these conversion factors.

25. What units should the answer be in? What value would you use to begin the problem and why? Solve the problem and show your work. Include all units and show cancellations of the units.
Prior knowledge needed for this activity:
- Need to know basic units of English measure.
- Need also to know basic algebra involving numerator/denominator cancellations and ratios.

Thoughts on presentation of the activity:
- You might want to model the fractional problem setup first shown in #13 below.

Target Responses for the tasks:

Critical Questions: (All values need either work or a short written explanation how you know)

1. How long does it take to drive 120 miles? 100 minutes, because 120 miles is the length of trip, and 1 trip also takes 100 minutes.
2. How long does it take to drive 240 miles? 200 minutes, because 240 is twice 120 (one trip) so the time is 2x 100 min.
3. How many miles can you drive on 5 gallons of gas? 120 miles, because 5 gal used in 1 trip, and 1 trip is 120 miles.
5. Show how you solved question # 4. Be sure to include the units in your calculations.
\[
\frac{120 \text{ miles}}{5 \text{ gal}} = \frac{x}{1 \text{ gal}} \rightarrow 120 \text{ miles} \cdot 1 \text{ gal} = 5 \text{ gal} \cdot x
\]
\[
x = \frac{120 \text{ miles} \cdot 1 \text{ gal}}{5 \text{ gal}} = 24 \text{ miles}
\]
6. Show the miles per gallon as a fraction (ratio). Which is the numerator? Which is the denominator? 24 miles is the numerator and 1 gal is the denominator.
7. Using a grammatically correct sentence describe how you made the choice for # 6. When units are listed as “x per y”, the “per” implies divided by, that means in “miles per gallon,” the miles must be divided by the gallons.
8. Is there another way to write the fractional relationship of gallons and miles (one that doesn’t show miles per gallon)? Show this way. \[
\frac{1 \text{ gal}}{24 \text{ miles}}
\]
9. Why might you want to write the ratio this 2nd way? Answers will vary. Possible answers: when solving for the term on the numerator (like how many gallons of gas are used when driving 68 miles). Also typically numbers larger than 1 are easier for people to visualize. For example the WWII battleship, *USS Missouri*, got an average of about 0.0066 miles per gallon, which is about 150 gallons per mile – most people find it easier to visualize 150 gallons and 1 mile than 0.0066 miles and 1 gallon (I have a mental picture of both the size of 150 gallons and the length of 1 mile; however, without doing a conversion to feet, I have no mental picture of how far 0.0066 miles is).

10. Here are 3 other ratio relationships that we can obtain from the model:

\[
\begin{align*}
\frac{1 \text{ bathroom break}}{120 \text{ miles}} & \quad \frac{5 \text{ gallons}}{100 \text{ minutes}} & \quad \frac{34 \text{ songs}}{1 \text{ hour}}\\
\end{align*}
\]

Write 4 other such relationships that you can obtain from the model, try to write ones that are not just inverses of ones already listed:

Answers will vary (any two items from the list in a ratio):

\[
\begin{align*}
\frac{1 \text{ break}}{100 \text{ minutes}} & \quad \frac{1 \text{ break}}{120 \text{ miles}} & \quad \frac{1 \text{ break}}{5 \text{ gallons}} & \quad \frac{1 \text{ break}}{34 \text{ songs}} & \quad \frac{120 \text{ miles}}{5 \text{ gallons}} & \quad \frac{120 \text{ miles}}{1 \text{ hour}}\\
\end{align*}
\]

These relationships are called Conversion Factors.

List the components that are necessary in a conversion factor: two values with different units that are equivalent to the same thing.

Using complete sentences consult with your group and come up with a description of a conversion factor. What are its essential components and what is its purpose?

A conversion factor is the ratio of two equivalent measurements, expressed with different units, and often different values, which are used to change the form of a value to an equivalent form with different units.

Box is to show Key Ideas

11. Which one of the conversion factors from #10 would you use to determine how long it would take to burn 12 gallons of gas? \( \frac{5 \text{ gallons}}{100 \text{ minutes}} \) or \( \frac{100 \text{ minutes}}{5 \text{ gallons}} \)
12. Construct the conversion factor needed to determine how many songs you would hear in 500 miles. Do not solve yet (you’ll do that in 13).

\[
\frac{34 \text{ songs}}{120 \text{ miles}}
\]

13. Solve # 12 mathematically. Show your work below and be sure to include units.

\[
500 \text{ miles} \cdot \frac{34 \text{ songs}}{120 \text{ miles}} = 141.67 \text{ songs}
\]

(that means you need to have 142 songs on your iPod® to not repeat songs)

Reflections:

14. As a group, write grammatically correct English sentences to describe the objective of the activity at this point. Be prepared to share your answer with the class. Answers will vary.

Possible answer: The purpose of this activity was to introduce, define, and practice using conversion factors to find equivalent forms of the same quantity

15. After having shared with the class, does your group still agree with your initial assessment of what the objective is? Answers will vary.

16. As a group, can you think of a situation when a scientist or chemist might need to use conversion factors to solve a problem? Give an example. Answers will vary. Possible answer: When the scientist has one form of a value but needs it in another form for some other purpose. Like when a graduated cylinder measures in ml but she needs it in liters.

Exercises:

Using conversion factors to solve a problem is called Dimensional Analysis.

You should now be able to solve the following problems.

17. Solve this problem without using a calculator:

\[
\frac{6 \cdot 12 \cdot 3 \cdot 13}{13 \cdot 9 \cdot 12} = \frac{6 \cdot 3}{9} = \frac{18}{9} = 2
\]

18. Write a mathematical rule that makes this problem easier to solve (you likely used this rule).

Terms common to both the numerator and the denominator of a fraction can be cancelled out or reduced.

19. Solve this problem:

\[
\frac{\text{miles} \cdot \text{songs} \cdot \text{gallons}}{\text{miles} \cdot \text{gallons}} = \text{songs}
\]

Use a cancellation line to solve the remaining problems.

20. How many miles would you have to drive to hear 43 songs? Show how you solve the problem using units and conversion factors.

\[
\frac{43 \text{ songs}}{1} \cdot \frac{120 \text{ mi}}{34 \text{ songs}} = \frac{5,160 \text{ mi}}{34} = 151.76 \text{ miles}
\]
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21. Using your answer from #20, how many minutes would this take? Again show how you solve the problem using units and conversion factors.

\[
\frac{151.76 \text{ miles}}{120 \text{ miles}} = \frac{100 \text{ minutes}}{1 \text{ minute}}
\]

\[
\frac{151.76 \times 100 \text{ minutes}}{120} = 126.47 \text{ minutes}
\]

22. Show how you can combine problems #20 and #21 into one. Draw a line through any units that cancel. Put your answer on the board.

\[
\frac{43 \text{ songs}}{1 \text{ song}} \times \frac{120 \text{ miles}}{1 \text{ mile}} \times \frac{100 \text{ minutes}}{34 \text{ songs}} = \frac{4300 \text{ minutes}}{34} = 126.47 \text{ minutes}
\]

23. Write a grammatically correct English sentence to describe which unit you will be left with in the answer. The final answer should be in minutes because both the songs and the miles cancel out.

24. The average human heart beats 72 beats/minute. If you live to be 80 years old, how many times does your heart beat? What conversion factors do you need to know to solve this problem. List these conversion factors. We would need to know the number of minutes per year in addition to the beats/minute. Since most people don’t know the yr→min conversion off the top of their heads, they’d need to break it into days, hours, and then minutes, so they’d need: 

\[
yr\rightarrow d \left(\frac{365.25 \text{ days}}{1 \text{ yr}}\right), \quad d\rightarrow hr \left(\frac{1 \text{ day}}{24 \text{ hr}}\right), \quad hr\rightarrow min \left(\frac{60 \text{ min}}{1 \text{ hr}}\right), \quad \text{and then min}\rightarrow \text{beats} \left(\frac{72 \text{ beats}}{1 \text{ min}}\right)
\]

25. What units should the answer be in? What value would you use to begin the problem and why? Solve the problem and show your work. Include all units and show cancellations of the units.

The units of the answer should be in beats. We need to start with 80 years because we want to convert a lifetime measured in years to an equivalent lifetime measured in beats.

\[
\frac{80 \text{ years}}{1 \text{ year}} \times \frac{365.24 \text{ days}}{1 \text{ year}} \times \frac{24 \text{ hours}}{1 \text{ day}} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{72 \text{ beats}}{1 \text{ minute}} \approx 3.0 \text{ billion (}3.0 \times 10^9\text{) beats}
\]